Sample D

Criteria	**Teachers' mark**
Personal Engagement	1
Exploration	3
Analysis	5
Evaluation	3
Communication	4
Total	16

Orbital Velocity and Orbiting Distance from a Center Mass

Introduction

In this lab we explore the relationship between the average distance of an orbiting body from the central mass and the average velocity of orbit; the independent variable being distance, and the dependent variable being the velocity of the body. For our purposes we will use the 8 planets and Pluto as the orbiting bodies and the sun as the central mass. The raw data will be collected from various online sources. From newton's universal law of gravitation and the laws regarding circular motion I hypothesise that the velocity will be related to the distance of the orbiting body from the sun. More specifically,

$$\mathrm{V} = \sqrt{G\frac{M}{r}}$$

Where, V is velocity, G is the gravitational constant, M is the mass of the central body, and r is the distance between the orbiter and the body it orbits.

Procedure

As mentioned before, raw data will be taken from online sources. In particular one will be interested in the average orbital radius and the average period. The data has been gathered from several different databases. The sources have been traced back, ensuring that data is not duplicated.

Sources

http://nssdc.gsfc.nasa.gov/planetary/factsheet/ - NASA
http://www.astronomynotes.com/tables/tablesb.htm - Nick Strobel
http://hyperphysics.phy-astr.gsu.edu/hbase/solar/soldata2.html - Kauffman

Data & Analysis

Table 1.1: Data from NASA		
Planet	Average Distance From the Sun [D] [10^6km]	Average Orbital Period [T] [Days]
Mercury	57.9 ±0.1	88 ±1
Venus	108.2 ±0.1	224.7 ±0.1
Earth	149.6 ±0.1	365.2 ±0.1
Mars	227.9 ±0.1	687.0 ±0.1
Jupiter	778.6 ±0.1	4331 ±1
Saturn	1433.5 ±0.1	10747 ±1
Uranus	2872.5 ±0.1	30589 ±1
Neptune	4495.1 ±0.1	59800 ±1
Pluto	5870.0 ±0.1	90588 ±1

Table 1.2: Data from HyperPhysics		
Planet	Average Distance From the Sun [D] [10^6km]	Average Orbital Velocity [V] [km/s]
Mercury	57.9 ±0.1	47.9 ±0.1
Venus	108.2 ±0.1	35.0 ±0.1
Earth	149.6 ±0.1	29.8 ±0.1
Mars	227.9 ±0.1	24.1 ±0.1
Jupiter	778.3 ±0.1	13.1 ±0.1
Saturn	1426 ±1	9.6 ±0.1
Uranus	2871 ±1	6.8 ±0.1
Neptune	4497 ±1	5.4 ±0.1
Pluto	5914 ±1	4.7 ±0.1

Table 1.3: Data from Astronomy Notes		
Planet	Average Distance From the Sun [D] [AU]	Orbital Period T [Days]
Mercury	0.387 ±0.001	87.969 ±0.001
Venus	0.723 ±0.001	224.701 ±0.001
Earth	1.000 ±0.001	365.200 ±0.001
Mars	1.524 ±0.001	686.980 ±0.001
Jupiter	5.203 ±0.001	4329.6 ±0.01
Saturn	9.537 ±0.001	10752 ±0.01
Uranus	19.191 ±0.001	30664 ±0.01
Neptune	30.069 ±0.001	60148 ±0.01
Pluto	39.482 ±0.001	90403 ±0.01

The data is formatted in varying ways, and thus, some tables require more processing than others (table 1.3 in particular). Provided are sample calculations for the calculations that occur this step.

AU to Km Conversion

An astronomical Unit is taken to be 149 597 871 km (Google)

Value	Uncertainty
∴149 597 871 [km/AU] * 0.837 [AU] = 57900000 [km]	∴0.001[AU]/0.387[AU] = 0.003
57900000[km]/10^6 = 57.9 [10^6 km] I 3 sig figs	0.003*57.9[10^6km] = 0.1 [10^6km]

57.6±0.1[10^6km]

Finding Velocity

Value	Uncertainty
V = 2πD/t	From before, 0.003
Days to seconds	∴ 0.003 + (0.001/87.969) = 0.003
t = 87.969[days] * 60[seconds/minute] * 60[minutes/hour]*24[hours/day] = 7600500 seconds I 5 sig figs	0.003 * 47.9 = 0.1
2π(57900000.[km]) = 363000000[km] I 3 sig figs (from before)	
∴ V = 363000000[km]/7600500[seconds] = 47.9 [km/s] I 3 sig figs	
V = 47.9 [km/s]	

V = 47.9±0.1[km/s]

Table 2.1 NASA: Average Distance Against Average Orbital Velocity			
Planet	Average Distance From the Sun [D] [10^6km]	Average Orbital Velocity [V] [km/s]	
Mercury	57.9 ±0.1	52	±1
Venus	108.2 ±0.1	38.06	±0.05
Earth	149.6 ±0.1	32.38	±0.03
Mars	227.9 ±0.1	26.22	±0.02
Jupiter	778.6 ±0.1	14.21	±0.01
Saturn	1433.5 ±0.1	10.543	±0.002
Uranus	2872.5 ±0.1	7.4225	±0.001
Neptune	4495.1 ±0.1	5.9415	±0.0002
Pluto	5870.0 ±0.1	5.1218	±0.0001

Table 2.2: HyperPhysics: Average Distance Against Average Orbital Velocity		
Planet	Average Distance From the Sun [D] [10^6km]	Average Orbital Velocity [V] [km/s]
Mercury	57.9 ±0.1	47.9 ±0.1
Venus	108.2 ±0.1	35.0 ±0.1
Earth	149.6 ±0.1	29.8 ±0.1
Mars	227.9 ±0.1	24.1 ±0.1
Jupiter	778.3 ±0.1	13.1 ±0.1
Saturn	1426 ±1	9.6 ±0.1
Uranus	2871 ±1	6.8 ±0.1
Neptune	4497 ±1	5.4 ±0.1
Pluto	5914 ±1	4.7 ±0.1

Table 2.3: Astronomy Notes		
Planet	Average Distance From the Sun [D] [10^6km]	Average Orbital Velocity [V] [km/s]
Mercury	57.9 ±0.1	47.9 ±0.1
Venus	108.2 ±0.1	35.00 ±0.05
Earth	149.6 ±0.1	29.79 ±0.03
Mars	228.0 ±0.1	24.13 ±0.02
Jupiter	778.4 ±0.1	13.074 ±0.003
Saturn	1426.7 ±0.1	9.650 ±0.001
Uranus	2870.9 ±0.1	6.8086 ±0.0004
Neptune	4498.3 ±0.1	5.4386 ±0.0002
Pluto	5906.4 ±0.1	4.7512 ±0.0001

Averaging Average Distance

Value

(57.9+57.9+57.9)[10^6km]/3 =
57.9[10^6km]

Uncertainty

(max-min)/2
(57.9-57.9)[10^6km]/2 = 0[10^6km]
∴take ±0.1[10^6km] as a standard

D = 57.9±0.1[10^6km]

Averaging Average Velocity

Value

(52+47.9+47.9)[km/s]/3 = 49[km/s]

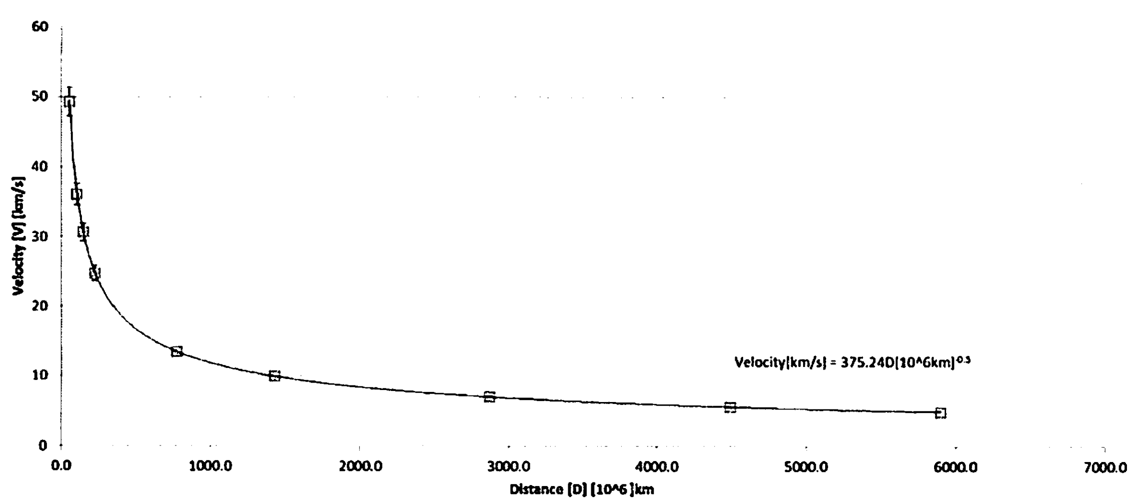
Uncertainty

(max-min)/2
(52-47.9)[km/s]/2 = 2[km/s]
But 49 + 2 = 51, Does not encompass 52. ∴ take 3

V = 49±3[km/s]

Table 3.1: Total Averaged Data		
Planet	Average Distance From the Sun [D] [10^6km]	Average Orbital Velocity [V] [km/s]
Mercury	57.9 ±0.1	49 ±3
Venus	108.2 ±0.1	36 ±2
Earth	149.6 ±0.1	31 ±1
Mars	227.9 ±0.1	25 ±1
Jupiter	778.4 ±0.2	13.5 ±0.6
Saturn	1429 ±4	9.9 ±0.5
Uranus	2871.5 ±0.8	7.0 ±0.3
Neptune	4496.8 ±0.9	5.6 ±0.3
Pluto	5900 ±20	4.9 ±0.2

Graph 1: Averaged, Distance Against Orbital Velocity



To linearize this data we will be using our hypothesis. It may seem a little backwards, but if it is linear then it serves as proof for our concept. Recall that,

$$V = \sqrt{G\frac{M}{r}}$$

If we put r into its own radical.

$$V = \frac{1}{\sqrt{r}}\sqrt{GM}$$

Thus, we will graph 1/root(r), the relationship should be proportional.

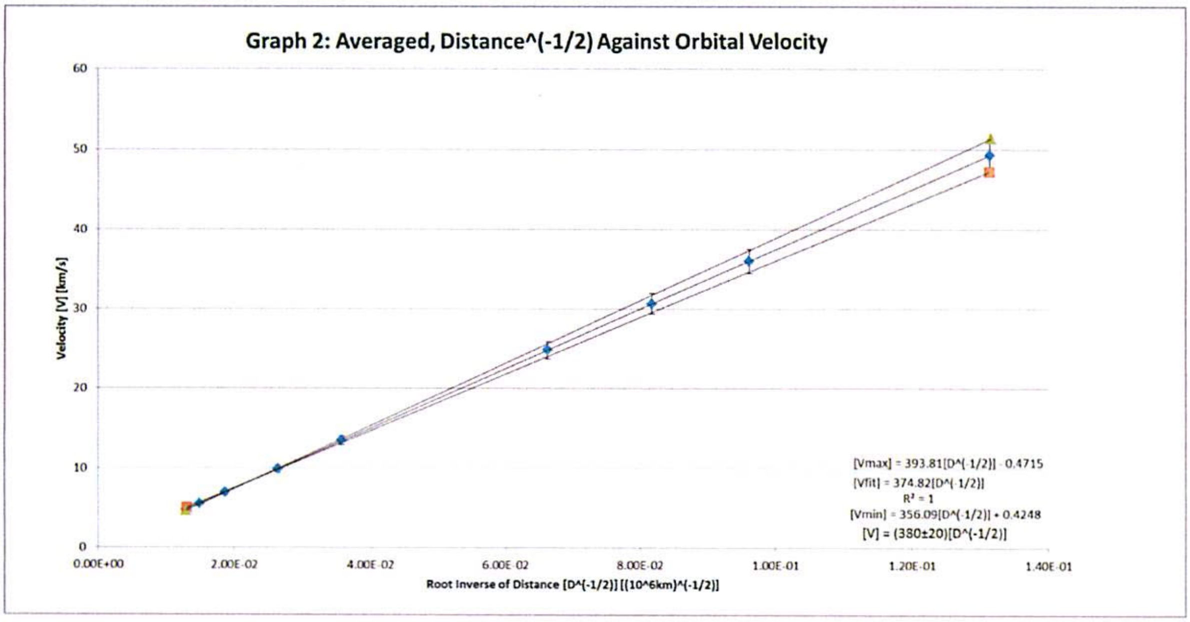
Value	Uncertainty
D^(-1/2)[(10^6km)^(-1/2)] = $\frac{1}{\sqrt{57.9[10^6 km]}}$	(0.1[10^6km]/57.9[10^6km])/2 = 0.00086
	0.00086[(10^6km)^(-1/2)]*0.131 = 0.00011

$\frac{1}{\sqrt{57.9[10^6 km]}}$ = **0.131[(10^6km)^(-1/2)]**

But, 3 sig figs so we take a standard of 0.001[(10^6km)^(-1/2)]

0.131±0.001[(10^6km)^(-1/2)]

Table 4.1: Linearized Averaged Data		
Planet	Average Inverse of Root of Distance [D^(-1/2)] [(10^6km)^(-1/2)]	Average Orbital Velocity [V] [km/s]
Mercury	1.31E-01 ±1.E-03	49 ±3
Venus	9.614E-02 ±4.E-05	36 ±2
Earth	8.176E-02 ±3.E-05	31 ±1
Mars	6.624E-02 ±1.E-05	25 ±1
Jupiter	3.584E-02 ±1.E-05	13.5 ±0.6
Saturn	2.646E-02 ±3.E-05	9.9 ±0.5
Uranus	1.8662E-02 ±3.E-06	7.0 ±0.3
Neptune	1.4912E-02 ±2.E-06	5.6 ±0.3
Pluto	1.30E-02 ±1.E-04	4.9 ±0.2



The average was calculated using the method shown before.

The final step would be finding the difference between the theoretical slope

$$\sqrt{GM}$$

And our experimental value 380±20[$10^3 km^{3/2} s^{-1}$]. But G is in $m^3\ kg^{-1}\ s^{-2}$, so we have to convert to $km^3 kg^{-1}\ s^{-2}$. To do this we divide by 10^9. ∴ G = 6.67384E-20 [$km^3 kg^{-1}\ s^{-2}$] (Google). The mass of the sun is 1.15214E30 [kg] (also Google).

($6.67384*10^{-20}[km^3kg^{-1}s^{-2}]*1.989*10^{30}$[kg]) = 3.643E5 [$km^{3/2}s^{-1}$]

$$\frac{3.643E5\left[km^{\frac{3}{2}}s^{-1}\right] - 380E3[km^{\frac{3}{2}}s^{-1}]}{3.643E5[km^{\frac{3}{2}}s^{-1}]}$$

The resulting value is -0.043, or a 4.3% deficit. The relative error for our experimental slope is 5.3%. Thus I conclude there is no detectable systematic error.

Conclusion

The relationship between distance from an orbiting center and the velocity of the orbiting mass is proportional, specifically:

$$V = \sqrt{G\frac{M}{r}}$$

I justify this by the data presented in graphs 1 and 2. As can be seen in both graphs, the best fit line goes through each of the points in or near it's center. Furthermore, graph 1 was linearized using the hypothesis stated. The resulting fit line on graph two proved not only to be linear (with an r^2 of 1) but also proportional to 1/root(D). Lastly, the small 4% difference between the experimental and theoretical value further supports the conclusion.